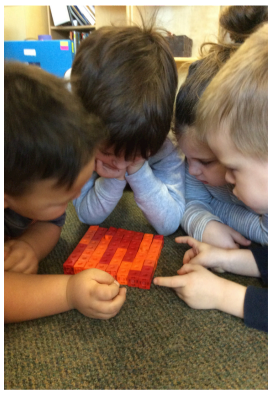
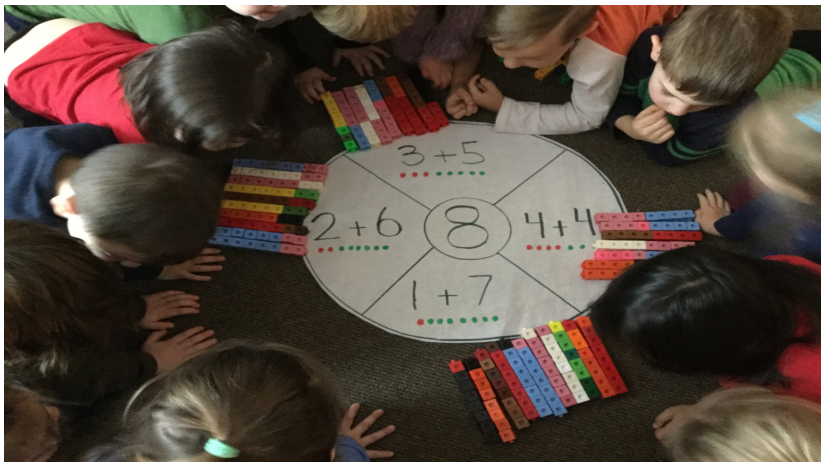
District School Board of Niagara

Math for Young Children
Lesson Study

June 12, 2018





Focus: Early Algebraic Thinking and Spatial Reasoning

M4YC Team: Mark Chubb (Instructional Coach), Mackenzie Condon (Trent research team), Susan Di Teodoro (Kindergarten), Tara Flynn (Trent research team), Petra Le Duc (Trent research partner), Melanie Otta (Kindergarten), Kristia Penlington (Kindergarten), Pam Schonewille (Grade 1), Kristin Willms (Instructional Coach)

Discussant: Dr. Cathy Bruce, Trent University

Educators (JK-Grade 1) from DSBN have been engaged in Lesson Study (M4YC) since December 2017, in partnership with the research team from Trent University, to explore

spatial reasoning and algebraic thinking.

We identified the following research questions:

- **How does algebraic reasoning begin / develop?**
- **What is the relationship between algebraic reasoning and spatial reasoning?**
- **What experiences support algebraic reasoning?**
- **What playful / spatial experiences encourage children to build algebraic thinking?**

AGENDA
10:30 am – 1:00pm

- Introductions and background provided by the planning team (1 hour)
- Research lesson - Kindergarten (30 min.)
- Debrief (and working lunch) (1 hour)

Debrief protocol:

i. Teachers who taught lesson
ii. Observations from M4YC planning group
iii. Comments from guests
iv. Discussant: Dr. Cathy Bruce

TMERC TRENT MATH EDUCATION RESEARCH COLLABORATIVE

TRENT UNIVERSITY

Ontario

Social Sciences and Humanities Research Council of Canada

Conseil de recherches en sciences humaines du Canada

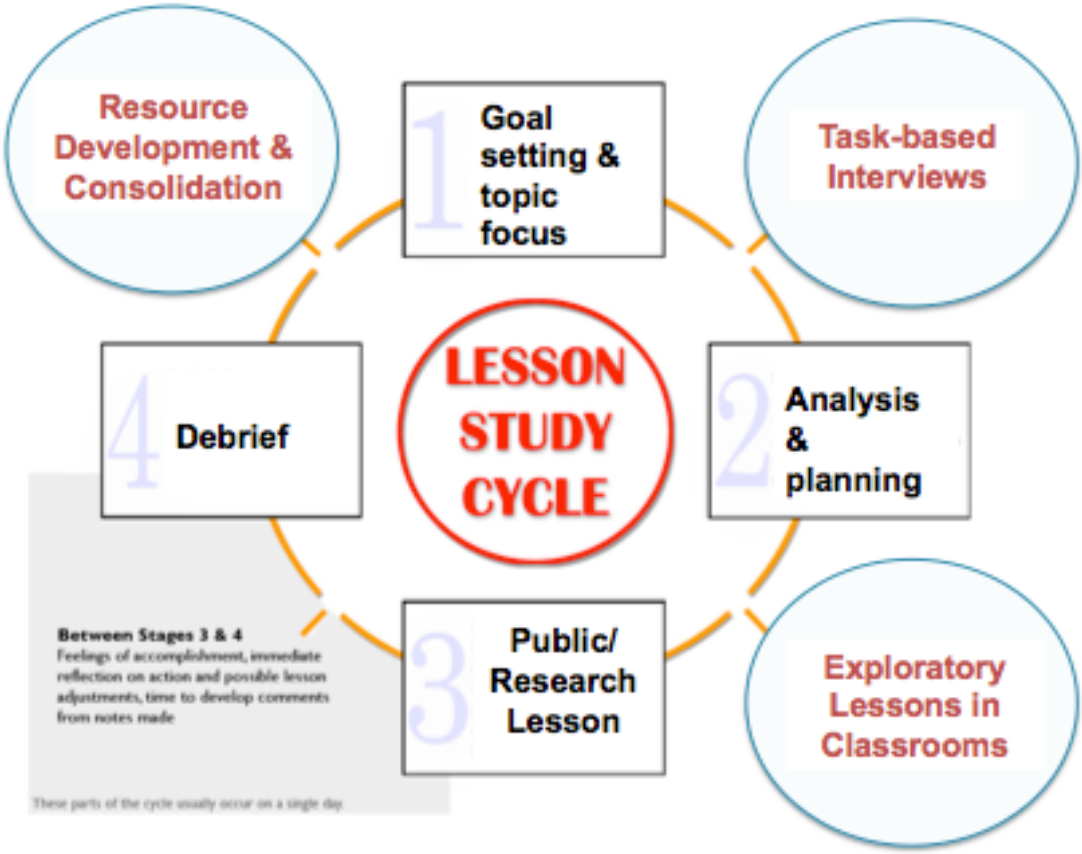
Canada

This research was supported by the Social Sciences and Humanities Research Council of Canada.

BACKGROUND

This team has been working on a Mathematics for Young Children Lesson Study cycle from December 2017 to June 2018.

The Lesson Study Process:



Why focus on math?

- Duncan et al. (2007, 2009, 2011) identified early math skills as best predictor of school success in math, language and social studies (very large scale studies)
- Math is a better predictor of language skills than early reading is …of later language skills!
- And math is a good predictor of overall credit accumulation (Ontario data)

Why focus on early years?

- The link between socioeconomic status (SES) and school success is well established.
- Low SES differences show up as early as age 3 (Blevin, 1996 & 2008; Lefevre et al., 2009)
- Without early intervention, children of low SES and/or with math difficulties will experience a “cascade of mathematics failure” from which it is extremely difficult to recover (Jordan & Levine, 2009).

Why focus on spatial reasoning?

- Spatial reasoning is the ability to create and manipulate mental representations of actual and imagined shapes, objects, and structures (Cohen & Hegarty, 2012)
- We are mobile organisms living in a 3-D world: “spatial intelligence has evolutionary and adaptive importance.” (Newcombe & Frick, 2010)
- Empirical evidence indicates that spatial imagery reflects not just *general intelligence* but also the ability to solve mathematical problems, especially *non-routine problems* (e.g., Casey, Nutall & Pezaris, 2001; Wheatley et al., 1994)
- Spatial ability is a predictor of success in STEM disciplines (Science, Technology, Engineering and Mathematics) as well as the Arts (Newcombe, 2012, 2013; Wai, Lubinski & Benbow, 2009)
- Three reasons to pay attention to spatial reasoning in mathematics:
    - Spatial thinking and mathematic thinking (and achievement) are strongly related
    - Spatial thinking is malleable and can be improved through education and experience
    - Schools play an important role in fostering spatial thinking

Why Playful Pedagogy?

- “Children do learn from play, but it appears that they can learn much more with artful guidance and challenging activities provided by their teachers” (Seo & Ginsburg, 2004)

CLINICAL INTERVIEWS

TASK 1: Cube Trains

Materials:

- 2 train lengths of cubes, one with 8 cubes and the other with 4
- We talked about either using all the same colour or having one train all one colour and one train all a second colour, to make it more visual and keep track of cubes being moved from one train to the other if that's a strategy that students use
- extra cubes available to students

Task Progression:

1. **"Are these equal?"**
2. **"How do you know?"**
3. **"What can you do to make these equal?"**
4. **"How could you test to see if you are correct/if they're equal?**"
5. *Could we also use the pan balance here? We initially only discussed it for the array question (see below) as an additional step/option for proving equality but I could see how it could also be used for the cube trains. Would they think of taking the trains apart and weighing the collections of cubes?*

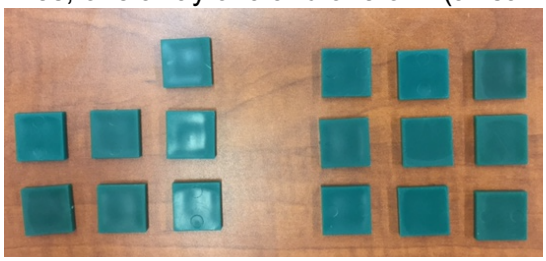
Look For/Listen For:

- Do students understand the concept of equality, or equal groups? What strategies do they use to create these (e.g., do they move 2 cubes from long train to short train, or do they add on or take away cubes to make them equal)?
- Are students counting each cube individually? Are students able to count on?
- Are students comparing lengths instead of counting?
- Are students adding, or subtracting for equivalence?
- Any evidence of rigid (a+b=c)/flexible operational (c=a+b)/relational operational (a+b=c+d) or comparative relational a+b=(a+1)+(b-1) thinking?

TASK 2a: Balancing arrays

Materials:

- Tiles, one array of 9 and one of 7 (all same colour)



- Pan balance
- Extra tiles available to students

Task Progression:

1. **"Are these equal?"**
2. **"How do you know?"**

3. **“What can you do to make these equal?”**
4. Encourage students to use the pan balance to explore equivalence: “**How could you prove to me?”** or **“How could you test if they’re equal?**”

Look For/Listen For:

- Do students understand the concept of equality, or equal groups? What strategies do they use to create these (e.g., do they slide one tile over, or do they take away one tile from the set of 9 to make them equal)?
- Are students counting each tile individually? Are they able to count on?
- Are students subitizing?
- Are students adding, or subtracting for equivalence?

Any evidence of rigid operational (a+b=c)/flexible operational (c=a+b)/relational operational (a+b=c+d) or comparative relational [a+b=(a+1)+(b-1)] thinking?

<u>TASK 2b: Balancing Arrays (Part 2)</u>

Materials:

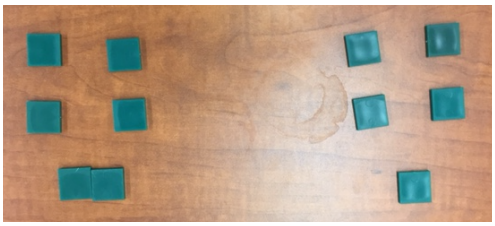
- Tiles (all same colour)
- See below for array set-up

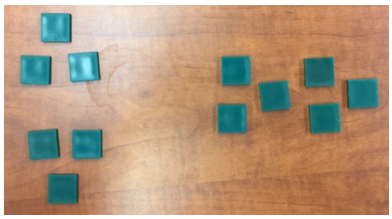
Task Progression:

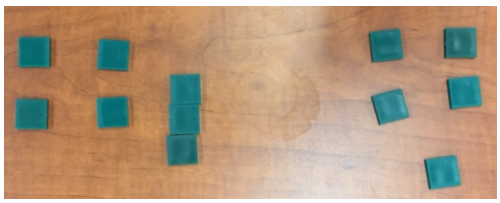
1. Set up first pair of arrays
2. **“Which one of these has more, or are they equal?”**
3. **“How do you know?”**
4. If arrays are unequal, “**How can you make these equal?”**
5. Repeat 1-4 for various arrays

Look For/Listen For:

- Do students understand the concept of equality, or equal groups? What strategies do they use to create these?
- Are students counting each tile individually? Are they able to count on?
- Are students subitizing? e.g., can they see/isolate the “four square” and compare the additional tiles?
- Are students adding, or subtracting for equivalence?
- Any evidence of rigid/flexible/relational operational or comparative relational thinking?
- How did the various arrangements affect student thinking or success?







TASK 3: Same – less – more with dot plates

Materials:

- Dot plate with standard array of 6
- 3 “thinking mats” with “same”, “less”, “more” (e.g., large cue cards with the word printed along the bottom, or plates with the words printed on them)
- round counters

Task Progression:

1. Show dot plate.
2. **“Can you show me an amount that is the same as this amount?”**
3. **“How do you know?”**
4. Now looking at the student’s collection (even if they are wrong in creating an equal amount, they will now use this amount as their new reference point): “**Can you show ma an amount that is less than what you have there?**”
5. **“Can you show me an amount that is more than what you have there?”**
6. If students are struggling to produce sets using the counters, they can be encouraged to use their fingers .

Look For/Listen For:

- Do students understand the concept of equal sets? What strategies are they using to create these? (e.g., visually, by creating the same array, or by counting)?
- Are students subitizing?
- Do students understand “less than”, “more than”? Is one easier than the other?
- What strategies are students using to create “less than” and “more than”? e.g, are they using the original quantity as a reference point and taking one away or adding one on to make different amounts without worrying about the exact number, or are they counting and trying to produce smaller and larger collections numerically?

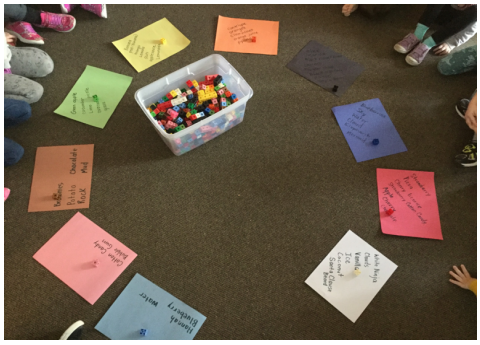
SCORING/OBSERVATION GUIDE

No sense of equality	Have a strategy visual comparing, counting, +, -, etc. or just saying what you know	Flexible more than one strategies	Relational moving one over to the other side to create equivalence (shows sense of relationship between numbers)	Other observations
Nicholas (compared lengths, added just 2 instead of 4, and then noticed that the 4 reds was the same as the 4 greens)	Sammy (compared lengths, counted then added) Kenna (compared, subtracted) Z’Khari (compared, added)	Zakai (added 4 and then subtracted 4) Cassandra (added 4 and explained how else she could have done it by breaking one of the trains at the 4th cube)		*Z’Khari might not understand what “equal means”*

EXPLORATORY TASKS

#1: Popsicle Factory

Learning in focus: composing and decomposing number, noticing/generalizing about number relationships, moving towards unknowns, spatial arrangements of number



Materials:

- Cube links, sorted by colour
- Outline of concept circle (on carpet?)
- Mystery bag

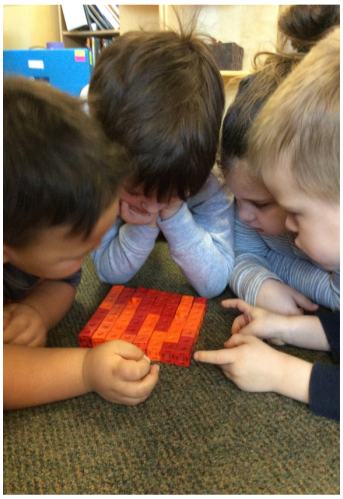
Progression:

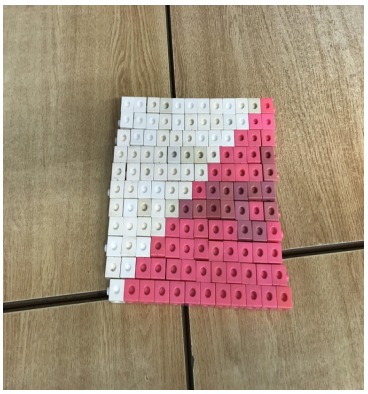
1) Free play exploration:
    - Choose flavours for each colour (whole group)
    - Explain the rules for making freezies: that all cubes of the same colour must be grouped together. Could have various examples and ask students which ones follow the rules and which ones do not.
    - Allow students to design their own freezies, as an exploratory free play
2) With parameters:
    - Explain to students that they need to make freezies that are 8 cubes long, using only 2 flavours [Grade 1's might use a larger length, such as 12]
3) Concept circle:
    - Have students (whole group) place their freezies on a concept circle, sorting the different combinations. There will be 4 options:
        - 1 of one flavour + 7 of another
        - 2 + 6
        - 3 + 5
        - 4 + 4
    - After the combinations are sorted, you may wish to label each section with a dot representation and number sentence (in addition to the cube links)

4) Mystery bag:
    - Show student a segment of a freezie with only one colour, ask how many of the other colour might be in the bag?
    - Show student a segment of a freezie with two colours, ask what other pieces might be in the bag? (both colours are an option, will have multiple possibilities)
5) Extensions:
    - Increase length of freezie
    - Increase # of flavours allowed
    - Explore cost of different sizes of freezies
    - T/F (True/Untrue) statements
    - Explore odd and even lengths of freezies

Look Fors/Listen Fors:

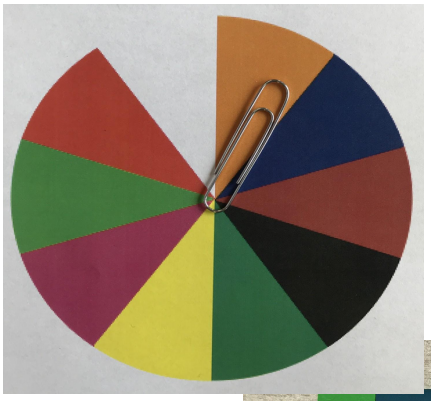
- What strategies do they use to create these (e.g., do they have a systematic approach)?
- Any evidence of relational thinking (e.g. subtracting cubes of one colour and adding equal amount of other colour)?
- Are students counting each cube individually? Are students able to count on?
- Are students comparing lengths instead of counting?
- Are students able to inhibit colour to figure out if 2 freezies are the same type?
- Any evidence of subitizing?

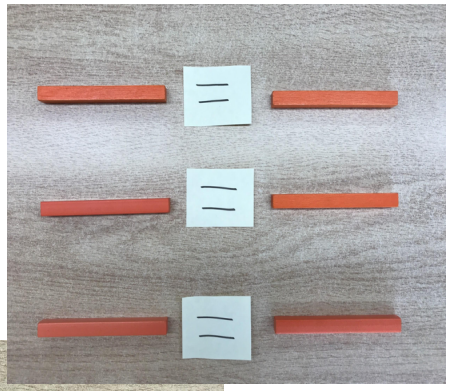


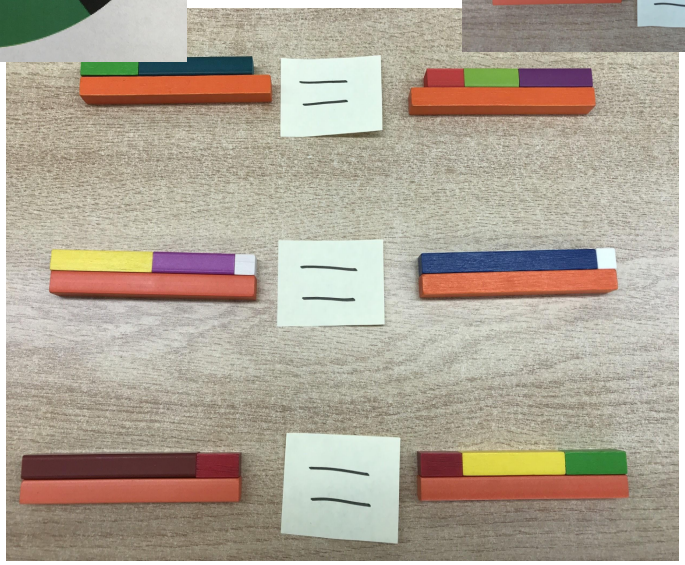


#2: The Train Game

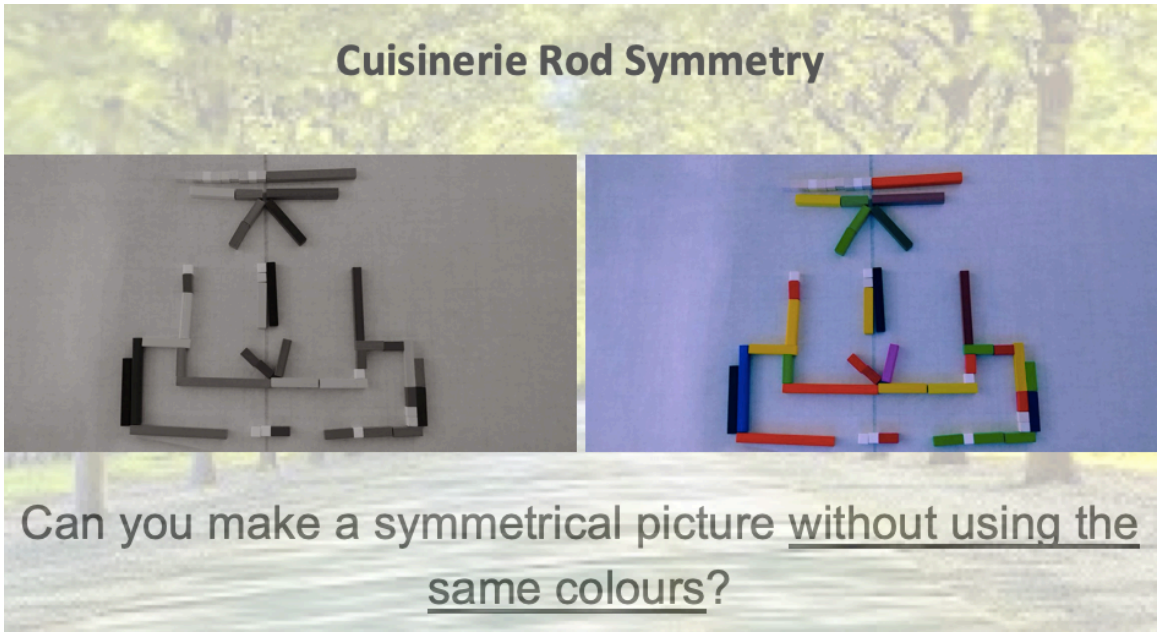
Learning in Focus: Exploring Equality with Relational Rods

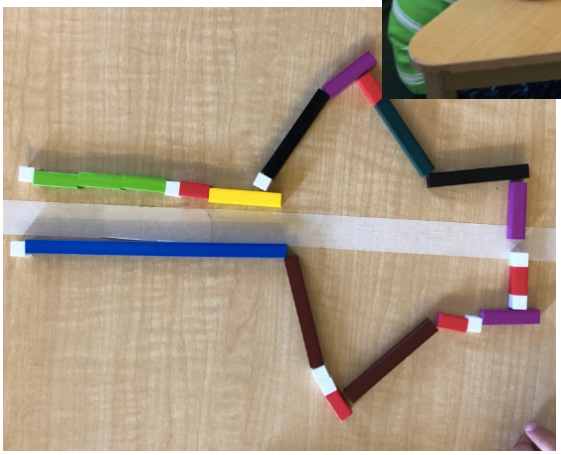


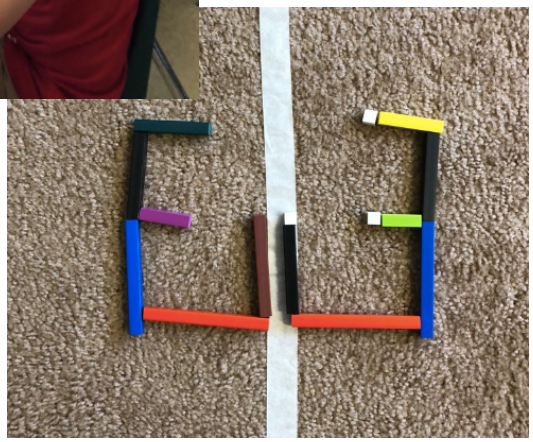




TODAY'S RESEARCH LESSON







OBSERVATION GUIDE 1:
SPATIAL REASONING

Look For	Notes
Spatial language used: direction, position, mirror, flip, same, different, bigger, smaller	
Gestures: Pointing, whole hand, full body, symmetrical, gesture, turning, rotating, spinning, balance	
Physical representations: What are they doing with the materials? How are they using materials to represent space, or number?	
Visualization Evidence that children are visualizing/using visual strategies?	

OBSERVATION GUIDE 2:
ALGEBRAIC THINKING

No sense of equality	**Have a strategy** (E.g., visual, comparing, counting, adding or taking away)	**Flexible** (e.g., more than one strategy)	**Relational** (Moving one over to the other side to create equivalence)	**Other observations**

<u>OBSERVATION GUIDE 3:</u>
OBSERVING THROUGH THE FOUR FRAMES

1. How do students collaborate/share responsibility/take on roles?
2. How do students deal with frustration?
3. Overall engagement with the task

BELONGING **(Belonging and Contributing)**	**ENGAGEMENT** **(Problem Solving and Innovating)**
EXPRESSION **(Demonstrating Literacy and Mathematics Behaviours)**	**WELL-BEING** **(Self-regulation and Well-being)**

OBSERVATION GUIDE 4:
MATH TALK

Consider making a word count for each of the words below and other math vocabulary for the students you are observing.

Examples	Tally	Key words or phrases
Transformation words (flip, rotate, slide, turn, etc)		
Location words (on top, beside, behind)		
Orientation or arrangement words (from the top, side, back, front)		
Symmetry words (same on both sides, same as the other half or other side, double, half, flip)		
Composing/decomposing/recomposing words (take apart, put together, put back together, this part goes into this part)		
Property words (edges, corners, colour, faces)		
Shape words (cubes, squares, rectangles, prisms)		

Other math language you hear: