St. Bernadette WCDSB
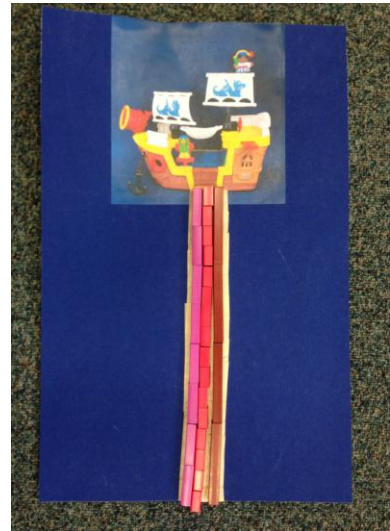
Math for Young Children Lesson Study

May 15, 2014

Research Question(s)

- How can we set up learning experiences for young students to discover principles of conservation in measurement?
- How can we connect experiences with linear, area and perimeter measurement to provide deep learning about measurement using standard and non-standard measurement tools?
- How can we foster precision in measurement?
- How do estimation and prediction strategies help students with not only measurement, but in building proportional reasoning strategies as well as number sense?
- How do students develop an understanding that: Area can be fixed but the shape of the figure can change, Perimeter can be fixed but the shape of the figure can change, and that area of the figure can stay the same while the perimeter changes?

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Trent University: Cathy Bruce, Tara Flynn, Sarah Bennett

Discussant: Janette Bobis, Associate Professor of Mathematics Education, University of Sydney

AGENDA

10:00-2:00:
– Introductions and background provided by the planning team
– Research Lesson
– Working Lunch
– Debrief: i. teachers who taught lesson, ii. observations from teacher planning group, iii. comments from guests, iv. Discussant

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BACKGROUND

This team has been working on a Lesson Study cycle from November 2013 through to May 2014. In this time, we investigated the curriculum area of measurement, particularly a "spatialized" approach to measurement.

The progression of our inquiry began with questions about proportional reasoning: how it spans the math strands and how it can help children use spatial reasoning to think about measurement.

Throughout this lesson study, we have explored length, area and perimeter. Many of our exploratory tasks have involved proportional reasoning and making connections between proportionally related units. We have considered how to encourage students to connect the spatial arrangements in measurement with non-standard units to numbers and standard units with precision.

WHY FOCUS ON MATH?
- o Duncan et al. (2007, 2009, 2011) identified early math skills as best predictor of school success in math, language and social studies (very large scale studies)
- o Math is a better predictor of language skills than early reading is …of later language skills!
- o And a good predictor of overall credit accumulation (Ontario data)

Early Intervention matters:
- o The link between socioeconomic status (SES) and school success is well established.
- o Low SES differences show up as early as age 3 (Blevin, 1996 & 2008; Lefevre et al., 2009)
- o Without early intervention, children of low SES and/or with math difficulties will experience a "cascade of mathematics failure" from which it is extremely difficult to recover (Jordan & Levine, 2009).
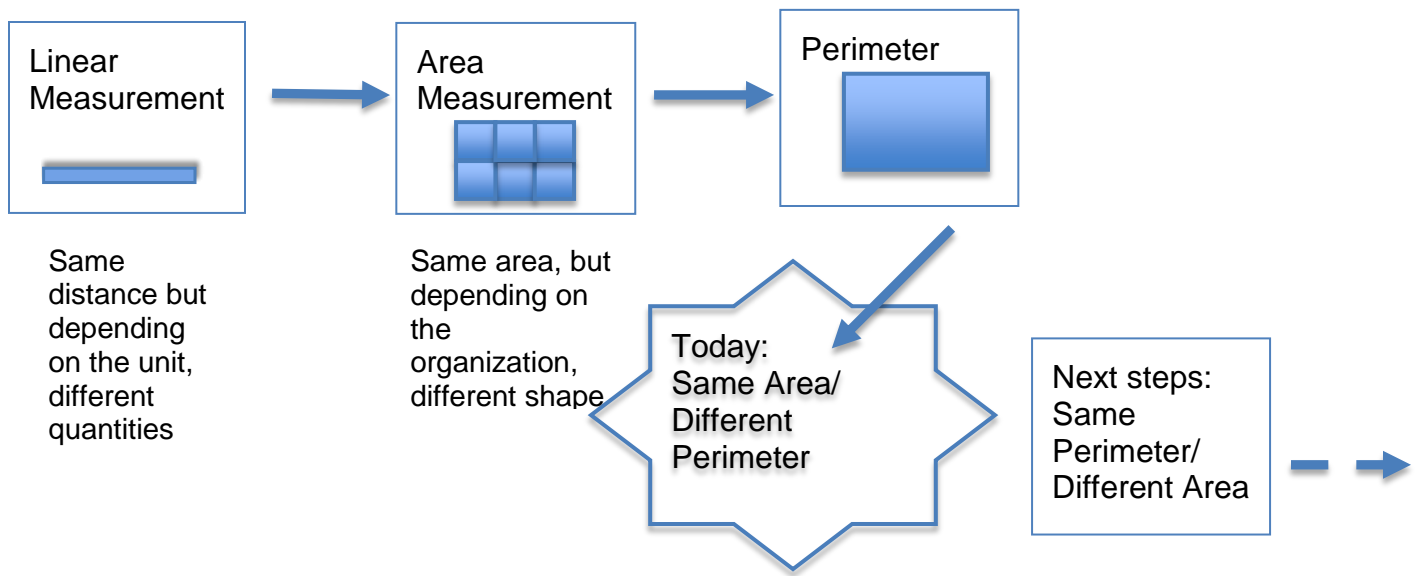
WHY FOCUS ON MEASUREMENT?

Some of the KEY Measurement IDEAS we encouraged through exploratory lessons were:
- **The importance of the unit used to measure (i.e., the smaller the unit, the bigger the quantity; the larger the unit, the smaller the higher the quantity)**

- **Conservation of area (i.e., shapes that look different can have the same area)**
- **Conservation of length, including perimeter (i.e., shapes that look different can have the same perimeter)**
- **The importance of estimating/predicting and the power of the "3 Vs": Visualize, Verbalize, Verify**
- **"Rules" of measurement: e.g., "no gaps, no overlaps"**

Our measurement trajectory:

Linear Measurement	→	Area Measurement	→	Perimeter

Same distance but depending on the unit, different quantities

Same area, but depending on the organization, different shape

Today: Same Area/ Different Perimeter

Next steps: Same Perimeter/ Different Area

CLINICAL INTERVIEWS

Selected students participated in one-to-one task based interviews with researchers. The interviews were video-taped and used for discussion with the team.

Tasks were:
- Number Knowledge (to assess basic number understanding)
- Peabody Vocabulary Test (to control for language)
- Key Math (number, geometry and measurement measures)
- Measurement tasks: number line tasks with missing values and proportional reasoning; determining length when object starts at different locations on the number line

These tasks will be repeated with the same students after the public research lesson day, to look for changes in understanding.

EXPLORATORY TASKS

We designed a series of tasks to explore our central research questions

Exploratory Task Set 1

Overview:

Conservation of Number Task
Materials/set-up
- 2 sets of cubes (5 red and 5 blue)
- Spread out cubes evenly on table (5 red above 5 blue)

Prompts & Actions
- Are there the same amount of red and blue? Or does one line have more than the other?
- Have student count to check/confirm that they do have the same amount
- Watch what I do [spread out red]
- Which one has more?
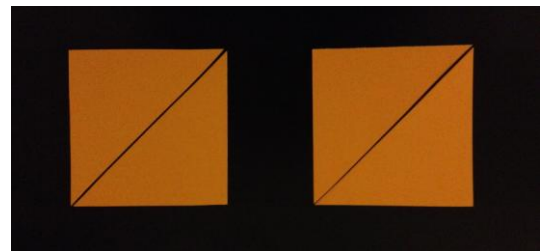- [Squish together red] Which one has more?

Conservation of Area Task
Materials/set-up
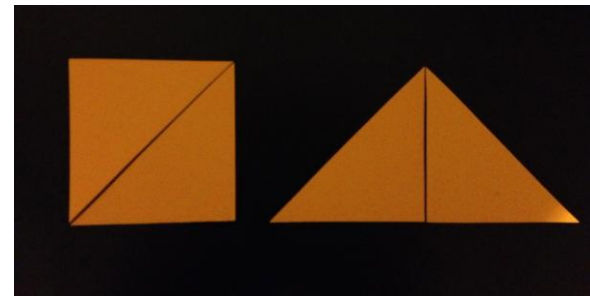- 2 squares cut diagonally into 2 triangles each, laid out side by side

Prompts & Actions
- Do these two squares take up the same amount of space?

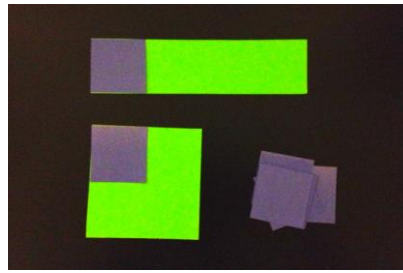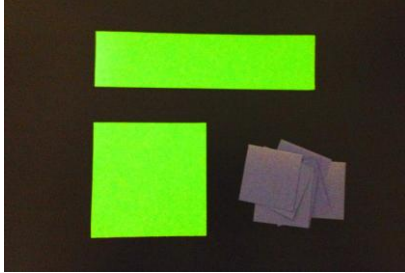- Watch what I do… [move the pieces of one square to make a large triangle]

- Are they still the same size / are they still taking up the same amount of space?

Fill-in Area Task
Materials/set-up

- one square (4 square units, 2x2) – colour 1
- one rectangle (4 square units, 4x1) – colour 1
- 6 square units available for student – colour 2

Prompts & Actions

- Here's a square…how many of these square cards would fit into this space?
- Estimate, then check by over-laying square units
- Clear off square
- Here's another shape [rectangle]…how many of these square cards would fit into this space?
- Estimate, then check
- Clear off squares again. Lay out both square and rectangle
- What do you notice? [then see if student notices that they are the same area] …how can they both be the same if they are different shapes?

"Does Size Matter?" Proportional Length Task
Materials/set-up

- Strip of a certain length
- Set of different tools to measure (e.g., Cuisinaire rods, wiki sticks or something else that can be made proportional to each other)
- Proportions 1:2:4 (whole-length, half-length, quarter-length)

Prompts/Goals

- How many of these would you need to measure this length? (Estimate, measure)
- Explore the different proportions – can students identify ratios (e.g., half)?

Goals: What do our students already know about measurement and what areas are challenging for them?

Observations:

JK/SK	Grade 1	Grade 2
Conservation of number: - Most students counted bears - Moving action was important (which bears moved and where) - SKs had greater success - Relying on spatiality (more to do with space than quantity) Conservation of area: - Attending to attribute of length (triangle is bigger) - "Triangles are always bigger than squares" - "You've done something different, but they're the same" Conservation of length: - Few said identical strips of paper, after being moved, were the same ("bigger because it's higher. You moved it up") - As soon as paper was moved, it became longer Fill-in area: - Most students estimated the rectangle would require more square-units to cover than the square Proportional length - Most estimates pretty close - Had to physically touch and move it to see	Conservation of number: - Very successful in one class - In another class, word "amount" seemed to be challenging Conservation of area: - Almost all said triangle is bigger/wider - If said they were the same, couldn't say why they were still taking up the same amount of space Conservation of length - 12 of 16 conserved length - "They're the same, you're moving one up, one over"	Conservation of number: - Some said more spread out row had more (vocabulary?) - In other class, all 2s got it (some needed to count, some saw just by looking) Conservation of area: - Most said the triangle was bigger because it was longer, wider, pointier - Nitpicky about them being the same at the start Conservation of length: - More students conserved length than area Fill-in area: - High estimates of number of square-units - Some didn't know how they could be the same amount if different shapes

Exploratory Task 2

Overview:
For this task we started with some common structures but varied contexts and materials.

Basic task:
Playful context involving "making 3 designs using 6 square units."

Rationale for making a finite number (instead of "make as many as you can/as many as possible"): Making "as many as possible" gets children to attempt to find the comprehensive set of possible "combinations"; it's about combinations and permutations rather than discovering area.

Common structures:
- Edges of units must be flush (no corners touching) – must be made in a grid formation
- Introduce the word area in relation to other ways of talking about area
- Playful context
- Square units: scale of the squares never changes and they match the context (the size of the square units proportional to the scale of the context, e.g., if designing something small for elves, will use small squares/square objects, but if designing a large area, use larger squares)
- Get students to discover the sameness of the area by checking/comparing their own and then comparing other student's using another object (e.g., cardboard squares)

Possible variations (materials/contexts):
- Foam pieces that could make a play area or puppy pad
- Learning carpet
- Pieces of white paper (snow fort base)
- Green lego base (building platform)
- Something with a grid to help them connect to the number
- Puzzle mat (could be tricky)
- Post-it notes
- Grid paper
- Think about whether you will get students to number the squares and/or use different colour squares (all six are different colours)
  - Designs have to use all the colours? Some think maybe not because then we are laying on another attribute (colour) that might be a distractor. Cathy S. might try to see if the students start to unitize 6 as one or if colour helps them to see the units
- Notion of using something transparent to overlay to check

Goals:
Inquiry question:
What playful contexts can we explore to increase the precision of measurement to move students towards more formal measurement?

BIG IDEA: Shapes can have the same area even though they look different (different shape, same area).

Observations:

JK/SK	Grade 1/2	Grade 1/2
Context: elf on the shelf needs a landing pad - All groups came up with 3 designs (12 altogether) - "Always the same if you have 6 sides" - Talked about area of 6 - Some fixated on perimeter; got tripped up on the word "area" - Attending to area, but sometimes that's getting mixed in with perimeter and length	Context: play pad for bay-blade - Looking to see if anyone made the same shape – orientation was a distractor - Thought long rectangle was bigger, despite also being made of 6-squares (longer = bigger)	Context: puppy play pen - Drew on graph chart paper - Saw that different shapes have the same area

Exploratory Task 3

Overview:

Variations on Big Bill and Little Larry (from Copley, 2004, "Measuring Experiences for Young Children", in *Teaching Children Mathematics*)

***FOCUS: Length and proportional reasoning - the size of the units matter***

Goals:
- Thinking proportionally with linear measurement is the focus
- When measuring length, the units/objects we use should all be the same length
- Longer units lead to a length measure that has less (quantity) – less unit repetitions
- Shorter units lead to a length measure that has more (quantity) – more unit repetitions

JK/SK

Materials/preparation:
- Pre-measure linear strips in two colours, one that is twice as long as the other (1:2 ratio, later versions of this activity could use different ratios such as 1:3, 1:4), measure the rug to fit 10 of the larger units and 20 of the smaller units
- Leave these out on table for a couple of days ahead of the activity so that children have the opportunity to play with them
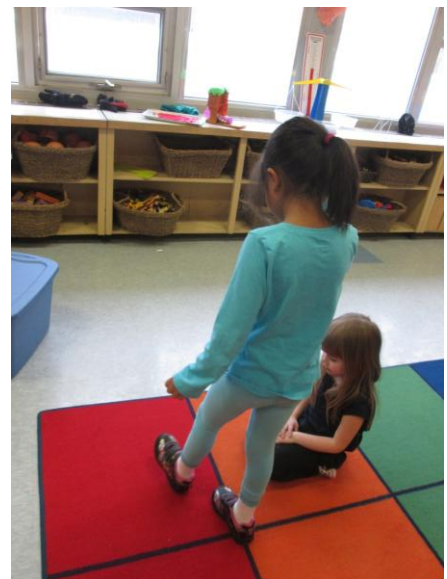
Scenario:
- "Mr. Dinner says the carpet is 10. Mrs. Hanley says the carpet is 5. What are they talking about? How could they both be right?"
- No parameters on tool selection, all materials are out, children can use different things (feet, hands, blocks, links, rulers, whatever is at hand)
- Children eventually will "find" the units that were used to measure (like a treasure hunt for the correct unit to measure the carpet)

Anticipated responses:
- Through exploration, students will find out that the carpet is 10 units long (longer unit) and 20 units long (shorter unit)
- We wonder if students will notice the relationship between the size of the units and the number of units (half the size, and twice the number)
- Students will come to a consensus that all units need to be the same in order to communicate the length

Key questions/language:
- "10 *what?*" "5 *what?*" – reinforce the importance of identifying/naming the unit
- Can they both be right? How?

Grades 1-2

Activity/Lesson #1

Problem Introduction:
We need to replace the area rug in our room. OR We need a new area rug for book centre
So I called the carpet store. And these two carpet people came on a Saturday. One was Big Bill and the other is named Little Larry.
Bill measured with his feet and then Larry measured with his feet. Here are the two measurements they made.
[Show two slips of paper with diagrams of same sized rectangles and numbers with length notation]

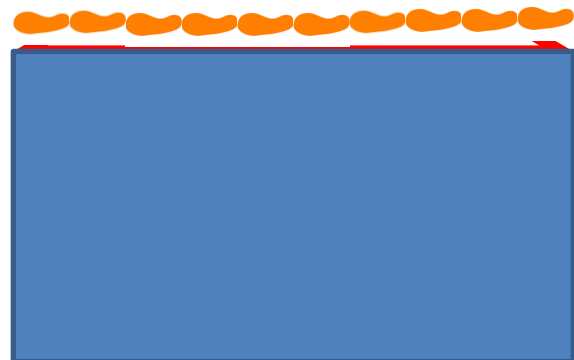*Dimensions on the rectangles:*
Big Bill – 5 feet x 10 feet
Little Larry – 10 feet x 20 feet

Materials:
Cut out construction paper feet for Bill (many copies – for manipulation on the taped carpet area and for folding, etc)
Note: Bill's foot should be 20 cubes long and Larry's should be 10 cubes long.
TAPE on floor to show carpet area – carefully matching the feet lengths.

Key Questions:
1a. Why did they give me different measurements?
- One foot bigger than the other
- Someone counted wrong!

1b. Which one has bigger feet?
- Some students may pick Larry because the numbers are higher
- Some students may reason that Bill has a larger foot because he doesn't have to take as many steps

2a. I asked the carpet measurers, "how did you measure it?" And Bill gave me this.
Hand out one of Bill's foot (construction paper) to each pair of students.
Key Question: What does this tell us?
2b. Key Question: If this is Bill's foot and it measured 10 feet for the length of the carpet, what do we know about Larry's foot?
- Discuss in pairs, etc.
- Halving the paper
- Cutting paper
- Take up their ideas

3. I used some cubes to measure Big Bill's foot. How many cubes long do you think his foot is?
- Estimate
- Check in pairs
- Discuss
Key Question: So, if Big Bill's foot is 20 cubes long, how many cubes long is Larry's foot?
- Hopefully students are saying things like: Big Bill is 10 cubes, and Larry is half that (5).
Or Double Larry to get to Bill
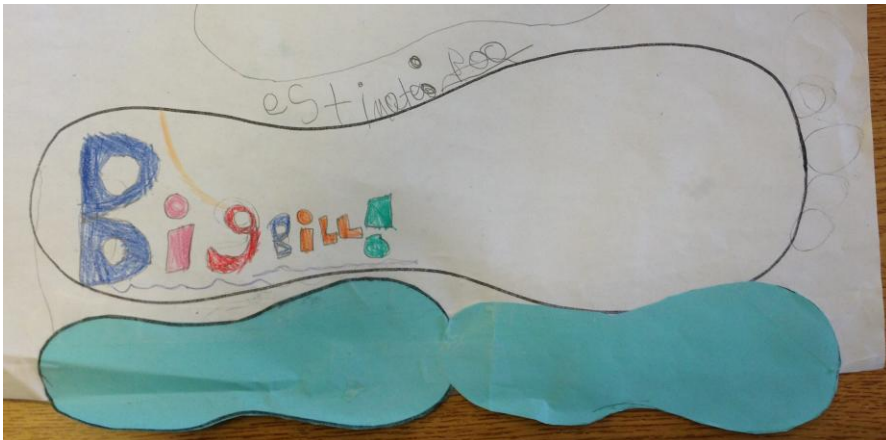
4. Key Question: If Big Bill's foot is 10 cubes long, how many cubes long should the area rug be?
- Bill's foot is 20 cubes, and the carpet is 10 feet long = 200 cubes

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At end of Activity 1 or at beginning of Activity 2

Explore length AND width
We spent a lot of time looking at the length of this area carpet.
Key Question: What can you tell me about the width?

Note – here are some other Brainstormed Possible Directions:

- Another foot comes along (different proportion) (1:3 or 1:4)
- Moving the feet to standard units (what can we do to make sure we get the right sized carpet)
- Selecting appropriate unit (What tool should we use for measuring a, b, c)
- Come back to area

Activity/Lesson #2

Goal: Beginning to think about Length AND Width (which tells us the AREA of the carpet)
Identify that we need to consider both length and width when measuring for a carpet (a rectangle has length and width)

Problem Introduction:
Last time, we were thinking about a fairly big area rug.
Now, we need a smaller mat! (for Mr. Dinner OR for the dog OR for getting snow off boots at the door of the classroom).
We need our carpet store to make the carpet.
Key Question: What size of carpet should we order?

- Go off in partners and talk about it, think about it, walk by the office
- Make a diagram on your chart paper (like the diagrams Larry and Bill made)

1. This might work: We need something about this size.
[Show a taped off area OR USE one chart paper cut to appropriate proportions]

Key Question: How can we measure this area to tell them how big of a mat we need?
- Take up student ideas about area, length, width

2. Give each pair a different unit length for their measurements.
Get them to measure the mat and report back.
Reminders of: Heel to toe (end-to-end measuring, no gaps, no overlaps)

3. Bring students back to report. There should be a range of answers because the students are using different lengths of objects as their units.

Uh oh! All of our measurements are different…. What are we going to do?
- Students talk about how they used different objects with different lengths
- Students compare their units to show they are different lengths
- Discussion about standards
- Have everyone measure with centicubes (everyone arrives at same length and width measures)

CONSOLIDATION: Make a chart of the findings about one or all of these.
What did we find out about:
   a. Measuring the little mat?
   b. Measuring a carpet / measuring a rectangle?
   c. The size of units for measuring?

Observations:

JK/SK	Grade 1	Grade 2
Context: built a road for fire truck instead of carpet - Used straws (full length and half length) - Photos (Ms. Hanley = 10; Mr. Dinner = 20) – they are both right. How is that possible? - Many different materials to explore - Challenge was to figure out what material was used to measure - SKs didn't think they could both be right (10 and 20) - Associating more (20) with longer unit – took a while to get that more doesn't mean bigger	Context: different path – hundreds day (how many steps would it be from our classroom door to the exit door? More than 100? Less than100? Exactly100?) - Some saw that if we use bigger object, it takes less (e.g. teacher's steps vs. students') Context: rug context – Big Bill had 10, Little Larry had 20, why do you think that would be? - Gave visual of feet – saw smaller foot was 20, big foot was 10 because it's bigger	Context: new carpet for classroom - Measured in "steps" (instead of "feet") - Measured length and width - Right away they saw that bigger feet meant less steps - Showed footprint for Big Bill – if this is Big Bill's foot and it took 10 steps, what do we know about Larry's foot (gestured splitting in half) - After using cubes, wanted to measure using "real numbers", i.e., a ruler

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Key Learnings for students/Key Learnings for us

Student Learning	Team Learning
Conservation of area: that area is the same regardless of arrangement Conservation of numbers: 5 is 5 regardless of arrangement Measurement concepts: where to begin, where to end, no gaps, no overlaps, edge-to-edge Need for one unit (one consistent measuring tool) If my unit/measuring tool is bigger, I'm going to need less, if it's smaller, I'm going to use more	Time to explore and talk…too often we are driven by need to fill curriculum and don't give time to talk (e.g., math-a-thon: extended time to think and talk and explore) Importance of vocabulary ("more than, less than, bigger, half, same as"): so their thoughts can be communicated clearly Importance of structures such as working in small groups to inquire, prove, disprove, working as problem solving teams Looking back at documentation: "what do we see now?" Revisiting the journey…allowing time to come back and build on, annotate their own work and further their thinking (e.g., looking at photos on IWB and typing in student observations) Richness of things in K that we never would have done before: - never would have explored *half*, *area* - wouldn't have occurred to me or I would have thought it was too hard (surprised with what they know) Connections across these experiences (e.g., "this is like when we made snowflakes and folded the paper in half") Importance of opportunity to collaborate with colleagues Exploration without a focus is a waste of time Importance of careful selection of materials

What do students need next?
Grade 2: area/perimeter
Grade 1/2: area
JK/SK: continue with linear measurement, continuing with concept of half, equivalence chart
(one long straw is equivalent to two shorter straws etc.)

Exploratory Tasks 4

Overview:
The team planned exploratory task #4 in two groups (JK/SK and grade 1/2).

JK-SK Group: Pirate Ropes

Context: Depth to a treasure (lengths of rope to get to the treasure or anchor); dive down from the surface of the water to the ocean floor/treasure (buried under the sand)

One long length (same as) Two short length
Fourths lengths (Melody & Heather) Double lengths (Cathy & Carol)

Boat – rope out of bottom of hull to ocean floor
As unit of measure: wiki sticks, rods – different coloured lengths
Wall in the hall – math talk in the hall (small group)

Goals of lesson:
-Find the buried Treasure!
-Precision of measuring a length
-Further develop concept of ratio (comparing the units): FIND A RULE
-Near and far predictions (estimating and verifying)

Materials:
Rods (cuisinaire) and stick tack or tape or magnets on bright link
Boat and rope to ocean floor (drawing on craft paper)

T-Chart could come from the narrative
KQ: What did you find out?
Ant. Resp: we need 12 red, the rope is 24 green lengths

KQ: So what is the rule? If I have one of these, how many of those do I need?
Ant Resp: For every blue piece I need 2 red lengths
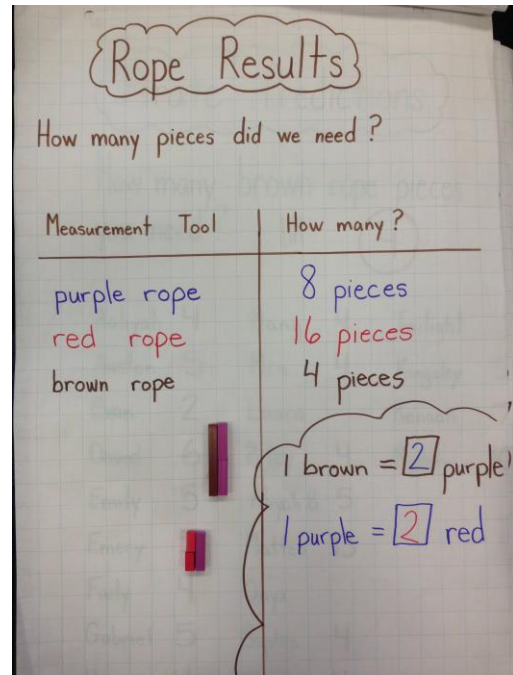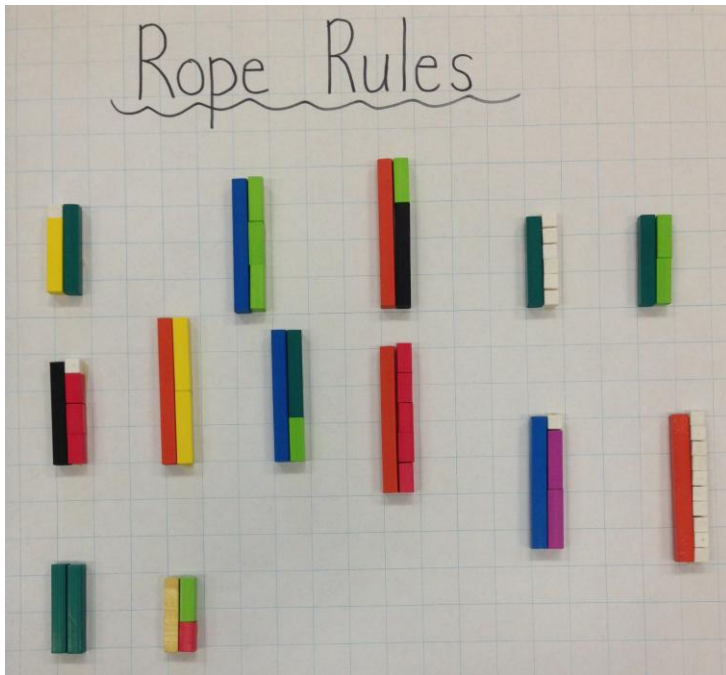One blue length is the same as 2 red lengths
One long straw is the same length as 1 short straws
Start making the T chart / symbols and cut outs (Petra template)

Then go back and do the third unit. (explore the third length, and record)
Look at this third length. I wonder how long that rope is?

KQ: The boat moved, longer rope, I wonder how long? With the different rods.

Rope Rules

Rope Results

How many pieces did we need?

Measurement Tool	How many?
purple rope	8 pieces
red rope	16 pieces
brown rope	4 pieces

1 brown = 2 purple

1 purple = 2 red

Other Possibilities:
1. Opportunities to prove by folding the paper rods in half to show half-length
2. Set up a PREDICTION board
3. Make your own rod. Take home paper rods and measure two things at home.

Grade 1-2 Group: Exploring Perimeter

Goal:
- experiences with perimeter (measurement of the outline of a shape)
- reinforcing the rules of measurement (no gaps, no overlaps, etc.)
- exploring standard units in realistic context (are you going to measure the fence in paper clips? No, you need to give a standard measurement)

Materials:
- picture of lego house (top-view only)
- put in middle of green construction paper

Context:
- X has built a lego house and wants to fence in the yard for the lego pets
- using the tools in your math toolkit, measure the perimeter
- measure the distance around
- record how long the fence would be
- focus on the fact that it's a bird's eye view – distance around
- which tools would you use and how many would it take to make a fence around the house (record all your possibilities)

Other possible contexts/extensions:
- picture frames
- go back to the area task (6 square units design)

Observations:

JK/SK	Grade 1	Grade 2
Context: Pirate Pete and Patricia need to measure rope to burried treasure - Students made fairly accurate Pirate Predictions, some very high - Made a chart of "rope rules", e.g., 1 brown = x purple - What could we measure with? "ladder, ruler, pencil, rope" - Vertical measure - Knew needed more of a smaller unit than bigger unit	Context: Lego character moving into a new house. Got two new puppies and needs a fence to keep them in - Master kit with standard and non-standard units - Used rules of measureemnt (following line, touching) - No one recorded measurements all the way around (plan to revisit ruler)	Context: Building fence to keep pets in yard - Estimated perimeter of their fence design - Used popsicle sticks, straws, cubes, measuring tape, ruler - Used 2 different tools - Justified choice of unit – e.g., paper clips because no overlap and clipped onto paper

Key Questions after Exploratory Tasks 1-4:
1. How do students transition from non-standard to standard units?
2. How can we encourage students to connect the spatial arrangements in measurement with non-standard units (like rods), to numbers and standard units (like number lines and rulers) with precision?
3. How do students transition from simple linear measurement to understanding of perimeter?
4. How can we encourage students to estimate and then compare their estimates to the actual measurement?
5. How can we map from here to standard number systems like number lines and rulers?

Exploratory Task 5: KRAZY KITES

Overview:
Research question:
How can we encourage students to connect the spatial arrangements in measurement with non-standard units (like rods), to numbers and standard units (like number lines and rulers) with precision?
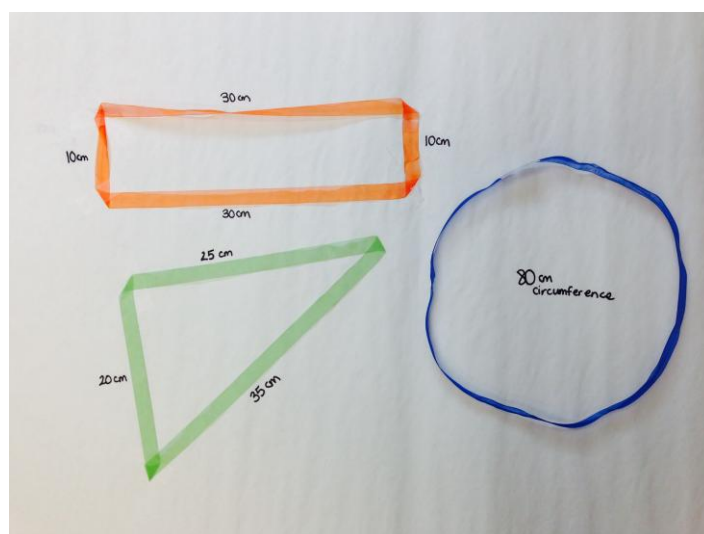
Goals:
1. To connect the concept of length and perimeter
2. Increase the accuracy of measuring
3. Connect measurement to standard units

Materials:
NOTE: We've suggested lengths, but please adjust as you see fit
- Not-too-stretchy string (maybe cotton kitchen string of different colours or ribbon that isn't too stretchy – that's what we used here)

- Three kite shapes: different shapes – triangle, circle, rectangle (all with 80cm perimeter)
- Rods, centicubes, rulers, meter sticks, created number line (using 10cm rod?)
- Tape
- Scissors

Scenario:
Here are three krazy kites.
Which kite do you think is bigger? (visualize, verbalize, verify)
- Students might refer to overall shape, length, width

Today, we're going to measure the PERIMETER (distance around an object) of the kites.
How could we measure the perimeter of these kites?
- Students might explore materials (or try to measure with ruler in the case of older students) and different strategies. Students may find this challenging (keeping track of length of different sides to get to a total perimeter length, and impossible in the case of the circle)

Introduce strategy of measuring with string (demonstrate)
Which kite do you think will have the longest perimeter?
- VISUALIZE (in your head…)
- VERBALIZE (discussion, why?)
- Then VERIFY – measure with string

What do we notice?
- All the kites have the same perimeter, but can be different shapes
- (Could recall with students how designs with 6 square-units had same area but all looked different)

Krazy Kite Rule: Kite tail length is the same as the perimeter of the kite

Let's verify to make sure all of our kites strings match our perimeter
- *Choice*: could have students make the tail and attach, or have the tail already attached (pre-measured to 80 cm, the same as the perimeter)

Now, we've been asked by Krazy Kites to design a new kite.
    Provide 2 pieces of string (60cm long)
    Remember rule: kite tail and perimeter are the same length
    Students use the one of the strings to make their new kite in *any* shape

The Krazy Kites Kompany needs information from you about how long to make their kite string and perimeter. How can we provide them with this information?

Options:

Connects to goals: - reinforces perimeter as length - Reinforces accuracy in measurement of

Point of exploration: Students may notice again that shapes can have same perimeter yet look

Connects to goals:
- Increase accuracy of measurement (using non-standard units)
- Assign numerical value to lengths
- Transfer non-standard measurements (string) to a measure with units (eg. Rods and building towards a number line arrangement), and from here to

16

- Measure using non-standard units (e.g., rods, centicubes, number line) OR standard units (ruler, meter stick), depending on the grade level
- Encourage students to relate length to a number (e.g., 6 orange rods, 60cm, 60 centicubes) so that instructions to the company are as precise as possible
- This part of the exploration could be done in many different ways depending on the students and needs of the class… we think there will be a lot of variation here, e.g., count with rods (JK), or use the rods to make a horizontal number line and label it (SK), use centicubes to make rods of 10 and transfer to number line (Grade 1), use a meter stick to measure (Grade 2)

Observations:

JK/SK	Grade 1	Grade 2
- Used 30cm perimeter (80 seemed too large for kindergarten – also matches up to ruler length) - Predicting bigger perimeter of cardboard shapes (triangle, square and rectangle) - More sides, bigger perimeter? - Longer/taller shape has bigger perimeter? - Minds were blown when all three shapes had same perimeter - "Every shape has a perimeter, the numbers are different" – JK - Perimeter is the "distance around an object" - First day used non-standard units, next day used rulers	- Three cardboard shapes (triangle, rectangle and circle) - A lot of students thought the circle would be bigger (placing circle on top, longer – looks bigger) - Measured with string – all the same (big 'aha' moment) - "I guess they can all have the same perimeter, but they can look different" - Revisited perimeter of rug and predicted how many feet (small feet, large feet) - Made accurate predictions (chose where to extend their learning and directed their own thinking from experiences) - Attending to length to predict perimeter - Given string to make their own shapes – all had different shapes, but same perimeter	- Powerpoint with kite images - Asked which kite is bigger? - Most said rectangle because it has more sides and was taller - One student asked what did you mean when you said "biggest"? - Then into perimeter – measuring with paper clips, paper rulers, string - All the same perimeter – connected to bay-blades/area (6-square units) task and talked about how all those designs looked different but had the same area - Built kites of their own using 60cm ribbon - Rules set by students – "no gaps, no overlaps"

Strategies used & conjectures made to estimate perimeter:
- More sides = greater perimeter
- Wider/taller shapes have greater perimeter (attending to one dimension)
- Overlaying shapes
- Dynamic strategy (mentally shift this shape to make it look more like that one, then compare)
- Visualizing
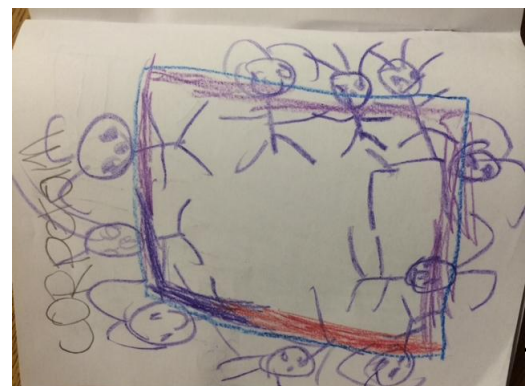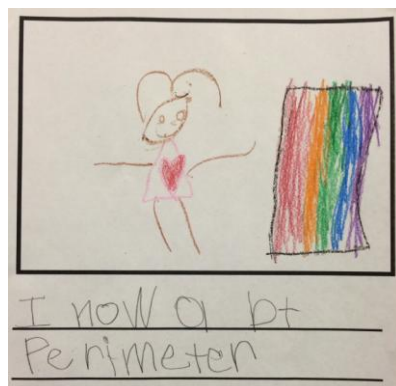
RESEARCH LESSON:

Background: In the exploratory lessons, students have explored different figures with the same area and different figures with the same perimeter. We are now interested in seeing how students approach a problem where figures with the same area have different perimeters. We thought a connection to Exploratory Task 2, where students made designs using 6 square units, would provide a good context and allow students to connect what they have already done to a new exploration.

Students: 8 Grade 2 students; 4 pairs

Materials:
Figures with a perimeter of 10, 12, 14
- 2x3 rectangle = perimeter of 10

- "b" = perimeter of 12

- "fancy w" = perimeter of 14

Tool kit with one yellow rod
Yellow rulers available
Envelopes with 6 squares (enough for a few for each group)
Glue or tape
Paper (for taping down figures and recording)
Pencils

Preparation:
Glue the three 6-square arrangements to pieces of bristol board (cut out along the edge of the figure)
Prepare tool kits and envelopes

Lesson Outline:
Opening:
Present three figures (with perimeters of 10, 12 and 14).
What do you notice about these figures?
Activate prior knowledge: All have 6 squares (remember back to earlier task with six squares).
"Lately we have been talking about perimeter, so
with your partner, order these figures from least to greatest perimeter."

In Pairs:
Visualize & verbalize

Anticipate students will ask for materials but ask them to just think about it. Encourage estimation (visualize & verbalize)

Whole group discussion:

Bring students together to compare their ordering of the figures with other pairs

Verbalize whole group

Key Question: How did you decide to order your figures from least perimeter to greatest perimeter?> What helped you decide the order of ther figures?

Anticpated student responses:

This shape is longer shape so it has a bigger perimeter.

They're all the same because they all have 6 squares.

This shape is taller so it has a greater prerimeter.

"Now, it is time to **verify**. Prove it!"

Give each pair of students a measuring tool:

Tool kit: one yellow rod (with yellow rulers available ONLY if students ask toward the end of the task) and have students determine the perimeter of each figure.

Whole group: What did you find out?

Anticipated responses: These figures have different perimeters but they have the same number of squares;

The number of sides they're sharing changes - some figures are more squishy in the middle;

This shape is just longer.

They all look different and they have different perimeters

"Here's a little envelope with 6 squares. Can you make another figure that has a perimeter of 14? You told me "fancy w" has a perimeter of 14."

Other figures with a perimeter of 14 include: "T"; "L"; 6x1 rectangle; cross; staircase

Rules:
1. **The sides have to be touching, not just the corners.**
2. **No gaps.**
3. **No overlaps.**

Glue down figures or photograph as you go. Ask students to verify – how do you know that has a perimeter of 14? Show me.

Anticipate that some groups may be able to find more than one figure with a perimeter of 14 (provide new envelope for each figure as they go).

Put photos or figures up on board. Look around at all the perimeters of 14 and see all the different possibilities.

IF TIME:

Ok, don't tell Mr Dinner, but we are going to break a rule. What if the sides do not have to touch, but they could just touch at corners. Would the perimeter be greater or less?

e.g.,
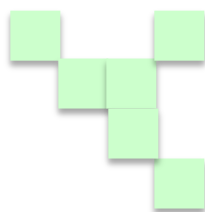
OBSERVATION GUIDE #1:
VISUALIZE, VERBALIZE, VERIFY!

- How are students developing an understanding that the area of different figures can stay the same ("6 squares") while the perimeter changes?

KEY POINTS OF OBSERVATION: *HOW DO STUDENTS USE **VISUALIZATION**, **VERBALIZATION**, AND **VERIFICATION** STRATEGIES TO ESTIMATE, PREDICT AND CONFIRM THEIR PREDICTIONS?*

NOTE SOME OBSERVATIONS FOR EACH.

VISUALIZE:

VERBALIZE:

VERIFY:

TEACHER REFLECTION:
WHAT COULD THIS LOOK LIKE IN YOUR OWN CLASSROOM? HOW DOES THIS LEARNING APPLY TO YOUR OWN STUDENTS?

OBSERVATION GUIDE #2:
MATH COMMUNICATION

- How are students developing an understanding that the area of different figures can stay the same ("6 squares") while the perimeter changes?

KEY POINTS OF OBSERVATION: *WHAT LANGUAGE AND GESTURES DO YOU SEE/HEAR?*

NOTE SOME OBSERVATIONS FOR EACH.

LANGUAGE:

GESTURES:

TEACHER REFLECTION:
WHAT COULD THIS LOOK LIKE IN YOUR OWN CLASSROOM? HOW DOES THIS LEARNING APPLY TO YOUR OWN STUDENTS?

OBSERVATION GUIDE #3:
MEASUREMENT STRATEGIES

- How are students developing an understanding that the area of different figures can stay the same ("6 squares") while the perimeter changes?

KEY POINTS OF OBSERVATION: *WHAT ESTIMATION STRATEGIES DO YOU SEE? WHAT STRATEGIES DO STUDENTS USE TO MEASURE AND COMPARE LENGTHS?*

NOTE ASSETS AND UNDERSTANDINGS AS WELL AS CHALLENGES:

ESTIMATION STRATEGIES:

MEASUREMENT STRATEGIES:

TEACHER REFLECTION:
WHAT COULD THIS LOOK LIKE IN YOUR OWN LEARNING ENVIRONMENT?
HOW DOES THIS LEARNING APPLY TO YOUR OWN STUDENTS?

OBSERVATION GUIDE #4:
GENERALIZATIONS, CONJECTURES & CONNECTIONS

- How are students developing an understanding that the area of different figures can stay the same ("6 squares") while the perimeter changes?

KEY POINTS OF OBSERVATION: *ARE STUDENTS ABLE TO NOTICE AND DESCRIBE THE RELATIONSHIP BETWEEN SHAPE (E.G., SIDES TOUCHING IN THE FIGURE) AND PERIMETER LENGTH?*

WRITE DOWN STATEMENTS YOU HEAR:

TEACHER REFLECTION:
WHAT COULD THIS LOOK LIKE IN YOUR OWN LEARNING ENVIRONMENT?
HOW DOES THIS LEARNING APPLY TO YOUR OWN STUDENTS?