



**St. John
WCDSB**



Math for Young Children Lesson Study March 9th, 2016

Research Question(s)

- What playful spatial challenges encourage children's knowledge of 'equal', 'same as' and 'balance'?
- Does providing students with a variety of tasks exploring the idea of 'same as or same but different' (through length, area, mass, composing shape, number/quantity) foster more conceptual understanding of the equality and build meaning of the equal sign?
- How do we continue to build on spatial reasoning tasks to include quantity relationships, expressions and equality?
- What investigative tasks provoke & evoke thinking about finding the unknown (missing part) in a balanced expression?

Teacher-Researcher Team:

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WCDSB: Petra Le Duc (Numeracy Consultant K-8)

LNS: Janice Baretto-Mendonca (SAO) and Wayne Brennan (SAO)

Trent Team: Dr. C. Bruce, Tara Flynn, Sarah Bennett, MacKenzie Condon

Discussant: Pat Milot, Pearson Facilitator
(former math consultant at DSBN)



AGENDA

10:00-2:00:

- Introductions and background provided by the planning team
- Research Lesson#1 – JK & SK
 - Working Lunch – Debrief Lesson#1: i. teachers who taught lesson, ii. Observations from teacher planning group, iii. comments from guests
- Research Lesson#2 - Gr. 1 & 2
 - Debrief Lesson#2: i. teachers who taught lesson, ii. Observations from teacher planning group, iii. comments from guests,
- Discussant

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BACKGROUND – OUR STORY

How did we get started?

This team has been engaging in Lesson Study since October 2013. In our first year of the project we explored various aspects of spatial reasoning through a geometry lens; our focus area was on spatial visualization within composing/decomposing/recomposing tasks. Starting in November 2014, we began to investigate the curriculum area of algebra, particularly a “spatialized” approach to the big idea of expressions and equality. The focus in year two of the project was aimed at building on our previous learning of composing/decomposing tasks to develop a better understanding of the equal sign as a statement of balance and “same as”. In our third year we have had the great fortune to partner with Dr. C. Bruce and Tara Flynn (principal researcher for the provincial project M4YC) to further build capacity of equality exploring multiple representations of equal, ‘same as’ in terms of properties/classification (i.e., length, area, number/quantity, shape composition, mass, distance) and beyond.

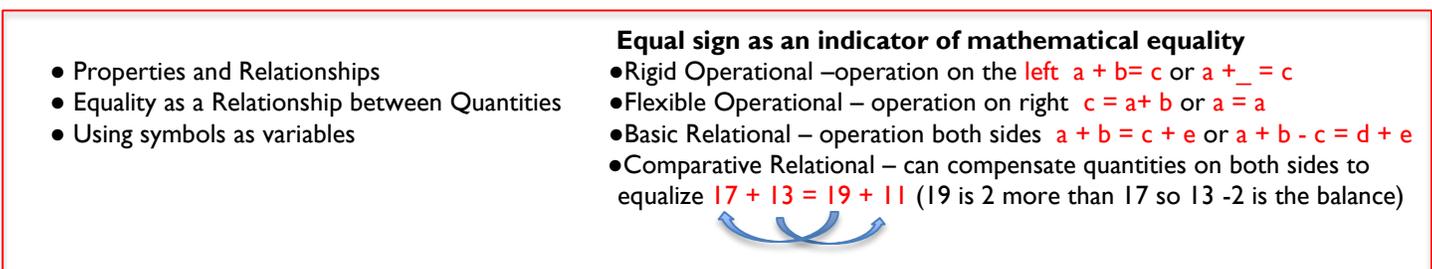
Aspects of spatial reasoning that we continue to deepen our understanding of throughout the three years of this project at St. John CES include:

(from IOSTEM working group on Spatial Reasoning, 2013)



Aspects of algebraic reasoning that we are learning about include:

(from Paying Attention to Algebraic Reasoning, 2013 & Children’s Understanding of Equality: A Foundation for Algebra, 2000)



Three Key Sources of Information for 2016:

1. Task-Based Interviews / Clinical Interviews
2. Literature we found useful
3. Our observations and thinking about ‘same as’ and equality over three years

TASK-BASED INTERVIEW (Team-created)

Engaging in these task-based interviews assisted us in developing exploratory lessons and informed our teaching of equality. A continuum of development of learning from JK to grade 2 also emerged through our observations of these tasks

Equality - Clinical Interview @ St. John CES, 2015-16

Student Name: _____

Grade: _____

- A. What does "same as" mean?
i. Can you show me an example?

What does "equal" mean?

- ii. Can "equal" mean something else?

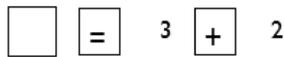
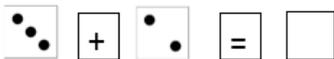
- B. Make something that is equal (use +, = tents and rods or pattern blocks) – document what they made



Grade 1 & 2 SET

Can you make these numbers sentences "equal", "balance" or the "same as"? (materials: dot plates/arrangements and white boards)

Tell: five is equal to five =

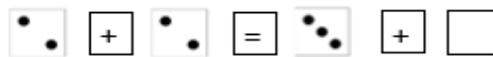
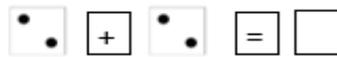
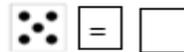


→Set up arrangements and white board

- C. Can you make these numbers sentences "equal", "balance" or the "same as"? (materials: dot plates/arrangements and white boards)

FDK SET

Tell: five is equal to five =



→Set up the rods, equal tents, bags and cookie props

- D. What's in the bag to make the number sentence "equal", "balance" or "same as"? (materials: equal tents, Cuisenaire rods, paper bags, cookies plastic ones if possible)

Tell: 'this green rod is', place it down...=' to this green rod'

1. =

2. = "What's in the bag?"

3. = "Now what's in the bag?"

4. = "What do you think is in these two bags?"

5. = "Now what do you think is in these two bags?"

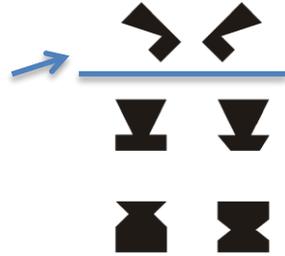
CLINICAL INTERVIEWS TASKS

Observing children in clinical interviews as provided us with a baseline of data, as well as identified the assets they already bring to their learning. Because we implemented the exploratory lessons across multiple grades, we have been able to identify continua of abilities and ways of thinking and to more precisely anticipate what students might do.

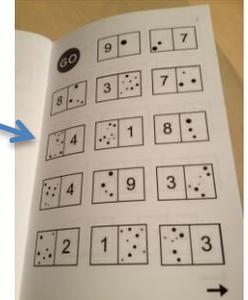
Clinical interviews with our students have involved three different tasks:

- 1) a magnitude test comparing symbolic and non-symbolic numbers and arrays (developed by N. Noseworthy);
- 2) a 2D mental rotation task (developed by Susan Levine);

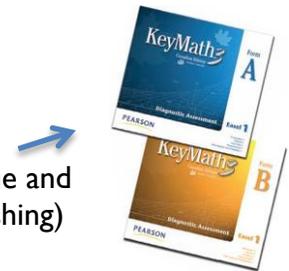
"If you put the two top shapes together, which picture will they make?"



"Choose which is larger or means 'the most'..."



- 3) a subtest of basic concepts in numeration: early number awareness; place value and number sense; magnitude of numbers; fractions; (developed by Pearson Publishing)



These tasks will be repeated with the same students after the public research lesson day, to look for changes in understanding.

LITERATURE WE FOUND USEFUL

Early Understanding of Equality By Aisling Leavy, Mairéad Hourigan, and Áine McMahon
2013 *The National Council of Teachers of Mathematics, Inc.* www.nctm.org.

Children's Understanding of Equality: A Foundation for Algebra By Karen P. Falkner, Linda Levi, and Thomas P. Carpenter
Mathematics
Teaching in the Middle School, copyright 2000 by the National Council of Teachers of Mathematics.

Measure for Measure: What Combining Diverse Measures Reveals About Children's Understanding of the Equal Sign as an Indicator of Mathematical Equality By Percival Matthews, Bethany Rittle-Johnson and Katherine McEldoon, Roger Taylor
Journal for Research in Mathematics Education 2012, Vol. 43, No. 3,

Paying Attention to Spatial Reasoning, 2014, Ministry of Education, Ontario

Paying Attention to Algebraic Reasoning, 2013, Ministry of Education, Ontario



OUR EARLY THINKING

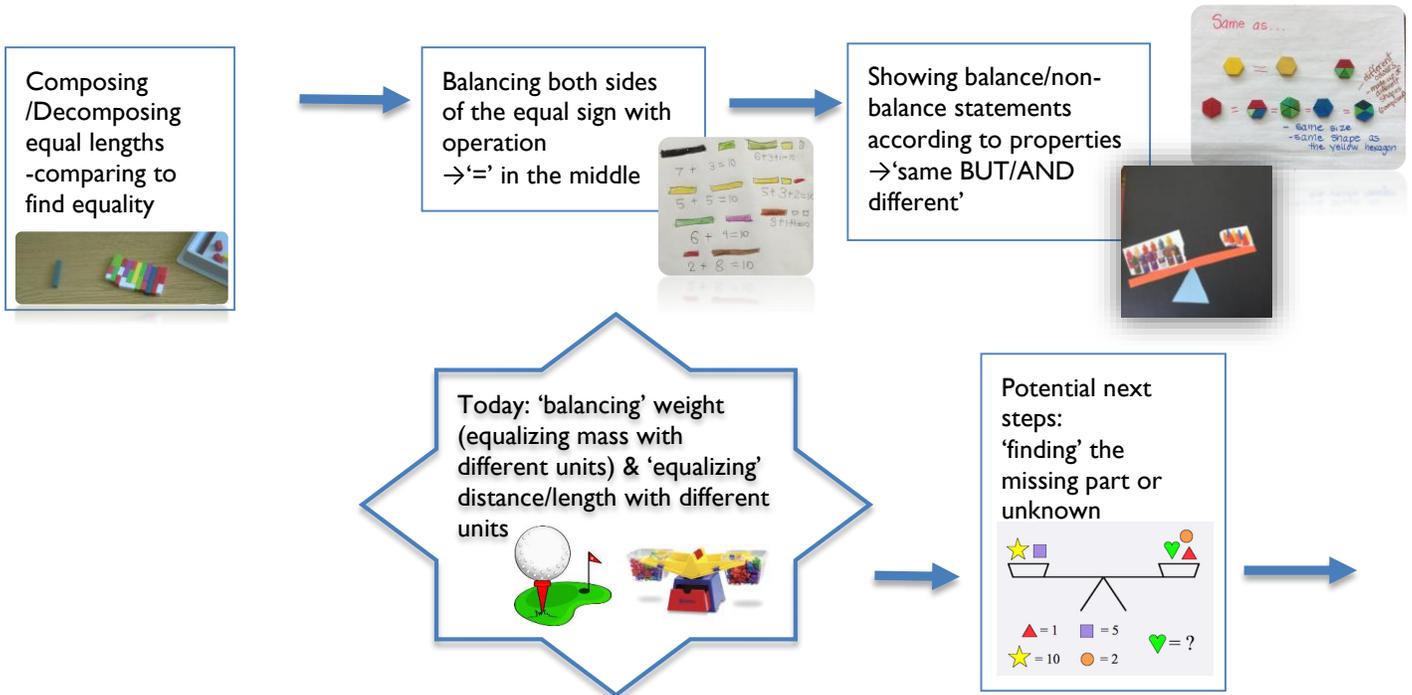
Students were having success with:

- comparing lengths (lining up different objects to measure equal lengths – no gaps no overlaps)
- recomposing an area with different sized shapes to keep the area equal (filling a puzzle shape different ways)
- using symmetry to test for same as (looks the same on one side as on the other)
- matching or overlaying as a strategy for conserving length, area, perimeter, quantity etc.

What we noticed that students struggling with:

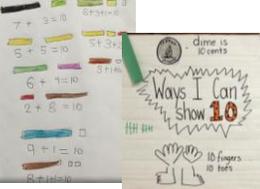
- using the equal sign as a ‘balance’ (placing the equal sign in the middle of two sides)
- thinking of equals as ‘same as in...’ and not just ‘the same’ i.e., matching images, objects, spaces exactly – generalizing the concept of equality
- understanding **how** two or more arrangements can be equal when we talk about the ‘same-ness’
- struggled to work backwards to find an unknown to create balance
- some students still had a very ‘rigid’ understanding of the equal sign ($_ + _ = _$ after the equal sign is where the answer goes)
- students understanding of equality was not as deep or generalized as we originally thought

OUR ‘SPATIALIZED’ EQUALITY TRAJECTORY 2015-16



EXPLORATORY TASKS

We designed a series of tasks to explore our central research questions

Task	Description	Concepts	Differentiation of the Task
<p>'tiny trains'</p> 	<p>Explore the properties of the Cuisenaire rods – conservation of length – 'same-length-as'</p> <p>Compose, decompose, and recompose lengths</p> <p>KQ: Can you make a train that is equal [in length] to this train?</p>	<p>-conserving length</p> <p>-making equal or "same as" statements with rods</p> <p>-measuring end to end (no gaps or overlaps)</p>	<p>FDK: Free play with just the smaller rods – dark green, yellow, purple, lime green, red and white.</p> <p>Grades 1 & 2: Free play with all rods – all colours</p>
<p>'rod riddles'</p> 	<p>Describe rod lengths by number quantity according to composition by colour and value - move from the models to the number values</p> <p>KQ: 'I am the 'same as' a green rod. What can I be?'</p> <p>'I am 'equal to' half of the orange rod. What could I be?'</p>	<p>-finding the unit value of each of the colours of the rods</p> <p>-comparing lengths</p> <p>-decomposing/composing equal lengths</p>	<p>FDK: Use a smaller selection of rods – start with the green and build from there</p> <p>Grades 1 & 2: encourage students to draw or write equality statements on balance scales.</p>
<p>'equal and operation tents'</p>	<p>Balancing the equal sign by positioning the '=' tent in the middle of the work space, students use manipulatives to make equal statements using '+' '-' and '=' tents</p> <p>KQ: Put the equal tent in the middle, can you make both sides equal using the operation tents?</p>	<p>-missing part or finding an unknown</p> <p>-placement of the equal sign</p> <p>-representing number and operation on both sides of the equal sign</p>	<p>FDK: set up challenges with '=' tent only</p> <p>Grades 1 & 2: use '+' and '=', using operation expression one side to start -moving to both sides of '=' with operations on both sides</p>
<p>'showing 10 in different ways'</p> 	<p>Decomposing/Composing a target quantity (10), representing 10 with a variety of models and with different combinations of rods.</p> <p>KQ: How many different ways can we show 10?</p> <p>KQ: How many different ways can you equal 10 using the rods?</p>	<p>-decompose the length of the 10 rod in different ways</p> <p>-conserving 10 in different ways</p> <p>-representing using an equations</p>	<p>FDK: showing 10 is different ways and contexts</p> <p>Grade 1 & 2: showing 10 using the rods</p> <ul style="list-style-type: none"> ○ using 3 addends (or multiple addends) ○ 2 addends on both side of the equal sign
<p>'Same BUT Different' tasks</p> <ul style="list-style-type: none"> ○ triangle/hexagon areas, & arrangements, dots, rods, candy jars, ○ 4 cube challenge ○ number line "=" task ○ Pan Balance, weight ○ vertical "same as" with rods 	<p>You can have the same number, length, area, weight, height, quantity, arrangement/shape of things but they might look different</p> <p>Different representations can be the 'same' ____.</p> <p>Introduce/reinforce language of properties where appropriate (remind students that we are sometimes focusing on different properties)</p> <p>KQ: (Have key point/outcome in mind) "so, what did we find out today?" "So what are you/we saying is the same BUT different...?"</p> 	<p>-equality is relative to properties/classifications</p> <p>-"=" sign can be used to express a variety of different equal situations</p> <p>-"same as in ____" can help explain the equal relationship</p> <p>Key take away: doesn't matter what the object is; doesn't matter if all the objects are same in a set when making an expression of equality</p> 	<p>FDK: using sets and objects that are similar but are arranged/composed in different ways</p> <p>Grade 1 & 2: using same objects in different ways, using different objects in same ways to show expressions of equality</p> 

*The entire lessons are fully available upon request email petra.leduc@wcdsb.ca

KEY LEARNING FOR OUR STUDENTS & KEY LEARNING FOR US

Student Learning	Team Learning
<p><u>Generalizations by grade</u></p> <p>Kindergarten:</p> <p>JK -practice was important (expanding that attribute descriptions)</p> <p>JK/SK- can recognize the '=' sign as a sign for balancing</p> <p>Some Sk's – use the '=' to make statements of equality on their own</p> <p>JK's – can fill in the blank when working with the '=' sign</p> <p>SK's could understand that things could look different but be the same.</p> <p>JK's did not see that so easily</p> <p>SK-starting to look past the surface classification or attribute and attend to more abstract attributes and classifications</p> <p>Grade 1 – clearly understood that things could be the same and different – getting better at describing (six year old view is same AND different is still merging)</p> <p>-use the '=' to show balance statements</p> <p>-getting more comfortable showing flexibility with the sides of the '='</p> <p>Grade 2 – quantity could be the same but length is different – speed and generalization – flexibility in thinking and the ability to articulate their assumptions/classification is increasing</p> <p>-very comfortable with using the '=' to show balance and 'same as'</p> <p>-becoming increasingly flexible with manipulating both sides of the '=' with addition as the operation</p> <p>Grade 1 & 2 (for both) – the more activities we do, the more they catch on – number line activity or conservation of the valentines hearts with capacity of jars, they are beginning to conceptually understanding statements of equality</p> <p>-they clearly understood that things could be the same but also different</p> <p>-they were getting better at being more specific (with attributes)</p> <p>-it is a huge leap to see that things don't have to be either same or different; can be both</p>	<p>-that equality is actually a very complex idea</p> <p>-we need to be precise in our language, by not simply asking if 2 things are equal but rather how they are equal →What classification or subsets are we looking at when we're looking for equality (i.e. Quantity, colour, shape, weight etc)?</p> <p>- equality is under pinned by classifications (thinking in a binary way) commonality you have to move away from attribute to classification</p> <p>-objects/materials/ideas may not be easily described as equal or not equal</p> <p>-things can be “the same but different” →It's important for students to be able to understand WHY things are equal, as opposed to simply memorizing addition and multiplication tables</p> <p>-there is a really important distinction between addition and multiplication as well →If you ask a student to give you 3 of something, they don't want to give you 3 <i>vehicles</i>, they want to give you 3 <i>cars</i>, however, 3 cars is actually related to <i>multiplication</i> →If you're speaking simply in terms of quantity, then objects don't need to be the same</p> <p>-a common problem is ensuring that students all start measuring equality from the same starting point. (i.e. with rods, having a fixed starting point)</p> <p>-it adds a layer of complexity if students are able to manipulate both sides of an equals sign or scale, as opposed to having one side fixed</p> <p>KEY POINTS:</p> <p>Scaffolding is important to help students layer their thinking</p> <p>Practice- students need more opportunities -young children need more practice, expanding on the attributes use of classification systems</p> <p>Observational skills – asking ourselves “are we missing out on something?”</p> <p>Flexible Thinking- asking students to think flexibly is a very big task. “Same but different” -complicated meaning of equality (dependent on experiences)</p>

DOCUMENTING OUR THINKING

Big Idea: We can train our brains.

erent

Number 5

Number 6

Number 10

Number 9

Learning Goal: We are learning how to show equal in different ways.

Success Criteria:

- ★ We can think about what we know about shapes and numbers.
- ★ We can take chances and try out our thinking to show equal with shapes and numbers.
- ★ We can talk about our work and our thinking.

How are these the same?
How are these different?

Same (same colour (green))

Different (different sizes (smaller/bigger))

Same triangle shape

Different (different colours)

Then we built the large triangle again using more pattern blocks (blue rhombus, red trapezoid)

Are these the same or different?

Same

- same size
- rods on right side are same as left
- same length
- 3 white = 1 green
- 2 yellow = 1 orange

Different

- diff #'s of rods
- diff colours

Some or Different?

Some - they both have 2 sides (one) (the dots are all light red) (they are all the same)

Different - different sized dots (they are different positions (1 on a dot) (same))

Some or Different?

Different - the yellow one has 3 points (blue one) 2 points (yellow)

Same - same shape (square) both have 4 sides (4)

Some or Different?

Same - both have pointers both have a point both have letters on them

Different - one is much larger - one is made of wood (one is yarn) - one is orange / one is yellow

Some or Different?

Same - both triangles both have 3 sides / made

Different - different sizes (1 on a dot) / # of shapes

Check Out Our Learning

Ways I Can Show 10

10 fingers 10 toes

1 dime is 10 cents

5 + 5 = 10

8 + 2 = 10

10 = 10

What is missing?

$10 = \square + 3$

$4 + \square = 10$

$1 + \square = 8 + 2$

$6 + 4 = 7 + \square$

I Can

- demonstrate an understanding of number relationships for numbers 0-10
- use, read, and represent whole numbers to 10
- investigate and develop strategies for composing quantities to 10
- investigate addition and subtraction in everyday activities

RESEARCH LESSONS

<u>Main activity</u>	FDK – “balancing bears” – comparing/relating the weight & size of different quantity of bears on a scale Grade 1 & 2 – “distance from the hole” –measuring the same distance in different ways with different sized units
<u>Students</u>	FDK – 6 JK & SK students (3JKs, 3 SKs) <ul style="list-style-type: none"> • 2 Co- lead teachers, support ECEs Grade 1 & 2 – 6 grade 1 & 2 students (3 grade 1s, 3 grade 2s) <ul style="list-style-type: none"> • One lead teacher, one supporting teacher (recording)
<u>Background:</u>	Leading up to these research lessons we have been exploring the ‘=’ as a symbol for balance, we have investigated representing ‘same BUT different’ relationships with a variety of items and we have composed and decomposed the same length, area, shape, quantities in different ways.
<u>Research questions</u>	What playful spatial challenges encourage children’s knowledge of ‘equal’, ‘same as’ and ‘balance?’ How do we continue to build on spatial reasoning tasks to include quantity relationships, expressions and equality?
<u>Criteria for both tasks</u>	<ul style="list-style-type: none"> • Playful explore of ‘balance’ and ‘equal’ • Opportunity for students to make unit statements about the size of unit and its quantity to measure something that is the same • Time for students to visualize, verbalize and verify their thinking together and on their own
<u>Materials</u>	‘balancing bears’ – bucket scales, 2 sizes of bears (small & large) <ul style="list-style-type: none"> • Prediction placemats & paper bears • Chart paper for co-teachers to record ‘same as’ statements (ratios) ‘distance from the hole’ – relational rods (brown, red, and purple rods); a few with magnetic strips on the back <ul style="list-style-type: none"> • For each group, strip of paper with golf tee and hole with a line drawn between the two (hash tags marking the beginning and end of the line). The distance should be exactly the length of 5 brown rods. • Chart paper, markers, whiteboard • Sticky notes and pencils
<u>Observation guides</u>	Relationships (1), Strategies (2) & Generalizations (3)
KEY IDEAS	Key mathematics of the task ‘balancing bears’: <ol style="list-style-type: none"> 1. The weight/mass of the unit affects the quantity (i.e., smaller bears there are more of them; larger bears require less 2. Bear weight/mass is relative and the relationship depends on the size of bear you are using to compare or balance the scale. 3. Recognize mass can be equal using different units (you can measure the same mass with different sized units) – “same and different” Key mathematics of the task ‘distance to the hole’: <ol style="list-style-type: none"> 1. The unit length affects the quantity (i.e., smaller units require more; longer units require less). 2. Unit lengths are relative and the relationships depends on the unit you are using (e.g., one brown = two purple; one purple = two red) 3. Recognize equal lengths using different materials (you can measure the same lengths with different materials) – “same and different”

RESEARCH LESSON: “BALANCING BEARS”

→Part I: Playful exploration to start

- **holding up one of each bears**
 - “Which one of these bears looks like the bigger/smaller bear?”
 - “Which one of these bears looks like the heavier bear?”
 - “If I had these bears on my ‘arm balance’ what would my arms look like?”



- **We are going to use the scale to verify/test to see which bear is heavier. See if it ends up looking the same as you were predicting.**

- “What if we moved the bears around would our scale still look the same? Would it look different?”

Key Questions:

- What did you do to make your scale to make it look like this? Is it the same or different from the friends around you? Did you use different size bears?
- What did you notice about the different sizes of bears and their weight/mass? Why is the one side lower than the other?
- How would you describe the way your balance looks?

PULL OUT THIS IDEA: the large bear is heavier, small bears are lighter - the heavier bear makes the scale lower on one side - the balance as “not balanced” or “not equal” because one side is lower than the other

- Clear the bears from your scale, let’s try a new challenge



→Part 2:

The next two challenges are about balancing or equal – **When the scale looks balanced or equal how does it look? Use your arms and describe how the scale will look when it is equal or balanced**

Challenge#1 – Make a prediction, **Visualize** (using the gesture of balance with hands) how many small bears would equal the ‘same weight/mass as’ one large/big bear

- using the paper bears make a prediction on your prediction mats

Verbalize –Could you show us using your arms what you predict about large bears and small bears?

- anybody who had the same idea/different idea as someone else?

Verify or test out your predictions using the bears and the scale

- paying close attention to how they measure with the scales



Key Questions:

- What did you notice when you tested your prediction? Was it the same as your prediction, close or not close? Did your thinking change?
- What did you do to your prediction to make it balance?
- How did you know it was balanced or equal?
- How did the sizes compare to each other?
- Was it hard to make a prediction?

Record their finding on the anchor (use a chart stand using pictures with fulcrum scale template) – record using the rows of bears

One large = 3 smalls

Challenge#2 – Make a prediction **Visualize** – using the gesture of balance with hands of how many small bears would equal the ‘same weight/mass as’ two large/big bears

- using the paper bears make a prediction on your prediction mats

Verbalize – turn to your partner and tell them what your prediction was and share your thinking– ask a couple of pairs to share their predictions

- anybody who had the same idea/different idea as their partner?

Verify or test out your predictions using the bears and the scale

- paying close attention to how they measure with the scales
- **Can you make your prediction mat match your scale?**

Key Questions:

- How did you know it was balanced or equal?
- What did you notice about the weight of 2 large bears and small bears?
- Was it easier or harder to make a prediction?
- How are these the same but different? (from part 1 to part 2)
- Do you see a connection between the 1st problem and the 2nd problem?
- Do you notice something happening on our chart?

Record their finding on the anchor (use a chart stand using pictures with fulcrum scale template) – record using the rows of bears.

Two large = 6 smalls

→**Super Challenge:**

On your prediction mat arrange the bears so that you have 9 small bears = 4 large bears on the other side – Your mat says: 4 large bears = 9 small bears

- Do you think this is true or false? **Visualize** the bears on the scale like they are on your mat
- Talk (**Verbalize**) to partner about what you are visualizing – whether you would need more or less **large bears** to make the scale balance

Verify – test it out – how did you make it balance?

- **What could you do to the large bears to balance the scale?**
-getting them to realize that they need to take away one large bear

DISCUSSION QUESTIONS (if time, feels appropriate)

What strategies did you use to visualize the bears balancing in your mind?

Was it hard or easy to make predictions about bears balancing?

What helped you think about balancing bears?

What was fun?

What was hard?

What are you most proud of?

If a friend were playing, what hint/tip would you give?

If you were to tell mom and dad what you did today, what would you tell them?

RESEARCH LESSON: “DISTANCE TO THE HOLE”

NOTE: Can use both “distance” and “length”; e.g., what is the *distance* from the hole to the tee? What is the *length* of the line?

- 1) Create a story for the children, such as, “My friend is building a golf course and she wants to know how far it is from the golf tee to the hole (start to finish).” Discuss the importance of end-to-end measurement! Demonstrate that it is incorrect to measure with gaps between rods, rods that are askew etc.



- **POINT OF OBSERVATION:** How did students build the first length? Multiple colours simultaneously; longer first, shorter after; did they do some careful fitting/replacing rods?
 - **KEY QUESTIONS & RECORDING:** Why do you think you have different numbers of rods?
 - **Listen for:** the longer the rod, the less you need; the shorter the rod, the more you need (write out this claim/axiom on a half sheet of chart paper)
- 2) Students will measure the distance with a string, or can simply use the paper strip as their distance to measure. They can use any rods they wish to measure the distance (except those colours that have been removed)
When complete, discuss the various results with students.
- **KEY QUESTIONS:** Why did Student A have one blue but Student B had 3 green? Why are the answers different?
- 3) Let’s assign colours to each student/group. One has brown rods only, one with purple, and one with red. Report findings [5 brown vs 10 purple vs 20 red], look at equivalence, same length but different units! Students might talk about doubling.

**BIG IDEA is that the unit length affects the quantity

- **KEY QUESTIONS & RECORDING:**
Start with brown, purple, red and record/make a table (written on a separate sheet from initial claim):

Distance of the first hole	
Rod	Amount
Brown (brown rod placed horizontally)	5
Purple (purple rod placed horizontally)	10
Red (red rod placed horizontally)	20

- Why do you think it took more red than brown?

- If I'm putting down 1 brown, how many purple would I need to make it equal? [Place rods on whiteboard] So now about one brown – how many red rods is 1 brown?

This is cuing up the next question; making generalizations about rod lengths – show on white board (using magnetic rods) Place rods vertically side-by-side to students can see the relationships clearly

- o Explore thinking, let the wrong answers come out
- o Students test out with rods
- Can revise thinking of original claim/axiom – adding comments about multiplication (e.g., purple is double the amount of brown; red is double purple; red is 4 times purple); Record new axioms on a new sheet of paper

4) Extend: My friend told me that the next hole is exactly 4 brown rods long, how many purple and how many red do you think I will need to make it equal? Predict and verify. Maybe use the whiteboard as a tool; create a chart

- RECORDING: Use sticky notes to make predictions about the number of purple and red and post up; once students decide on a number, write the amount up on the chart (with predictions up on the side)

Distance of the second hole

Rod	Amount
Brown	4
Purple	
Red	

IF TIME: We can't check this right now, but my friend wants the third hole to have a length of 10 brown rods? I wonder how many purple it would be? How many red it would be?

- Students make predictions about the number of purple and red

Distance of the third hole

Rod	Amount
Brown	10
Purple	
Red	

*Future extensions, or higher grade levels might use: 1 blue=3green=9white, or 1 orange=2yellow=5red

OBSERVATION GUIDES FOR BOTH LESSONS ON THE FOLLOWING 6 PAGES

**OBSERVATION GUIDE I:
RELATIONSHIPS ('balancing bears')**

Relationships	Examples of words or phrases and/or actions
Are the students connecting the relationship of big bear mass to little bear mass 3:1? How do they talk about this relationship?	
What other relationships are students attending to? (height/length of the bears, small & large)	
What kinds of comparison language are the children using? (e.g., heavier, lighter, more, less, bigger, smaller, add and take away, equal, same, balanced)	

Did you hear or see evidence of students using relationships to verify and articulate their thinking?

**OBSERVATION GUIDE I:
RELATIONSHIPS 'distance from the hole'**

Relationships	Examples of words or phrases and/or actions
Are the students noticing any relationships between the lengths of rods and the amounts used when measuring distance?	
What other relationships are students attending to? (space, size, intervals)	
What kinds of comparison language are the children using? (e.g., more, less, bigger, smaller, add and take away, equal, same, balanced, unit size)	

Did you hear or see evidence of students using relationships to articulate or verify their thinking?

**OBSERVATION GUIDE 2:
STRATEGIES ('balancing bears')**

Strategy	Describe...evidence of strategy...
<p>How are the children using their bodies to think about balance (embodied cognition)? –e.g., whole body versus hand weighing</p>	
<p>Does the paper representation help students to think about mass/quantity?</p>	
<p>Are the children paying attention to the exactness of the balance and the quantities, placement of the bears? (left and right buckets, but also placement within the buckets)</p>	
<p>Are the students using information from the charts to think about the ratio of big bears to little bears?</p>	
<p>Did the children use the three V's to help them make sense of the challenges?</p> <p>Are children changing their thinking as the challenges progress? And the verification progresses for each challenge?</p>	

**OBSERVATION GUIDE 2:
STRATEGIES ('distance from the hole')**

Strategy	Describe...evidence of strategy...
How did students build the first length?	
<p>Are students using multiple colours simultaneously?</p> <p>Are students using longer rods and then shorter rods as they move towards the end point?</p> <p>Did students do some careful fitting/replacing rods for exactness?</p>	
How are students using gesture to unitize or determine length? (finger spacing, indirect/direct measuring)	
When students are not directly measuring (e.g., hole number 3), what mathematics are they using to make predictions?	

**OBSERVATION GUIDE 3:
GENERALIZATIONS ('distance from the hole')**

<p>What mathematics are children using to make their predictions? Are they generalizing?</p>	<p>Are the mathematical structures (i.e., table of values, physical ratio proofs with the rods) supporting students in their math thinking?</p> <p>Are they helping students make sense of the mathematics?</p>

Are students applying generalizations when predicting about the golf holes that are different lengths (i.e., doubling from hole 1 to hole 3)?

In response to what we have just observed what might the next exploratory task include?